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Natural Convection from Isothermal Plates Embedded in Thermally Stratified Porous Media

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Nomenclature

A = constant, Eq. (4)
f = dimensionless stream function, Eq. (6)
G = auxiliary function, Eq. (14)
g = acceleration due to gravity
h = local heat-transfer coefficient
K = permeability of porous medium
k = effective thermal conductivity of porous medium
Nu = local Nusselt number, hx/k
q = local heat flux
Ra = local Rayleigh number, $Kg\beta(T_w - T_{\infty,o})x/\alpha\nu$
T = temperature
ΔT = temperature difference between the wall and porous medium, $T_w - T_\infty$
u,v = x and y velocity components
x,y = vertical and horizontal coordinates
α = thermal diffusivity
β = coefficient of thermal expansion
ϵ = dimensionless vertical distance, $Ax^\lambda/(T_w - T_{\infty,o})$
η = dimensionless horizontal distance, Eq. (5)
θ = dimensionless temperature, Eq. (7)
λ = constant, Eq. (4)
ν = kinematic viscosity
ξ = nonsimilarity variable
ϕ = auxiliary function, Eq. (14)
ψ = stream function

Subscripts

iso = quantity related to the uniform porous medium
o = leading edge of the plate
w = wall
∞ = ambient

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Introduction

NATURAL convection in porous media has recently received considerable attention for its important applications in many engineering problems. However, most previous studies have considered only a uniform environment. A thermally stratified field, although encountered frequently in many applications, has received rather little attention.

The first attempt to study natural convection in a thermally stratified porous medium was made by Johnson and Cheng.[1] Based on a general approach, they reached the conclusion that a similarity solution is not possible for the case of a stable stratification. Later, El-Khatib and Prasad[2] presented a numerical study on the effects of thermal stratification on natural convection in a horizontal porous layer with localized heating from below. Their results showed that the overall Nusselt number decreases with an increase in the thermal stratification. Recently, Nakayama and Koyama[3] extended the Johnson and Cheng study to report similarity solutions for that special case in which the ambient temperature increases inversely with the distance (i.e., $\Delta T \propto x^\lambda$, $\lambda<0$). It is felt, however, that such similarity solutions have limited application in practice due to the assumed temperature profile.

It is the purpose of this Note to extend previous studies by considering a more realistic temperature distribution which may be actually encountered in applications, e.g., ambient temperature varying with the distance ($\Delta T \propto x^\lambda$, $\lambda>0$). Since it has been pointed out by Johnson and Cheng[1] that similarity solutions are not possible for this case, series solutions and local nonsimilarity solutions are sought instead. It is expected that the solutions thus obtained will have useful applications in practice and will serve as a complement to the existing solutions.

Analysis

Consider a vertical plate embedded in an infinite porous medium (Fig. 1): The wall temperature is uniform at T_w, and the ambient temperature varies with distance from the leading

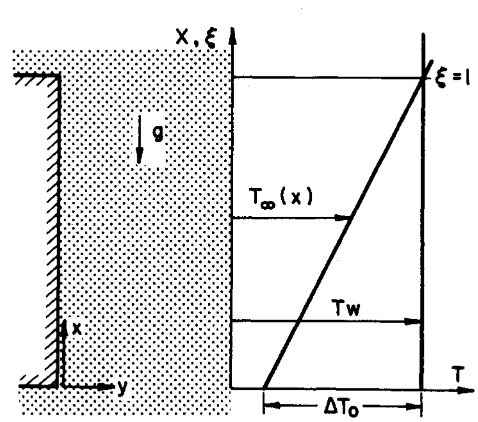

Fig. 1 Vertical plate in a thermally stratified porous medium.

edge. Having invoked the boundary-layer and Boussinesque approximation, the governing equations in terms of the stream function are given by

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{Kg\beta}{\nu}\frac{\partial T}{\partial y} \tag{1}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{2}$$

The appropriate boundary conditions are

$$y = 0, \qquad v = -\partial\psi/\partial x = 0, \qquad T = T_w \tag{3}$$

$$y \to \infty, \qquad u = \partial\psi/\partial y = 0, \qquad T = T_{\infty,o} + Ax^{\lambda} \tag{4}$$

where $T_{\infty,o}$ is the ambient temperature at $x = 0$. Positive values of A imply a stably stratified porous medium, the only concern in the present study.

Equations (1) and (2), together with the boundary conditions (3) and (4), are solved by the series expansion and local nonsimilarity method.

Series Expansion

We define a nondimensional normal coordinate by the relation

$$\eta = \left[\frac{Kg\beta(T_w - T_{\infty,o})x}{\alpha\nu}\right]^{1/2}\frac{y}{x} = Ra^{1/2}\frac{y}{x} \tag{5}$$

and a series for the stream function and dimensionless temperature, respectively, by

$$f(\epsilon,\eta) = \frac{\psi}{\alpha Ra^{1/2}} = f_o + \epsilon f_1 + \epsilon^2 f_2 + \ldots \tag{6}$$

$$\theta(\epsilon,\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty,o}} = \theta_o + \epsilon\theta_1 + \epsilon^2\theta_2 + \ldots \tag{7}$$

where f_i and θ_i are functions of η, and $\epsilon = Ax^{\lambda}/(T_w - T_{\infty,o})$. It is clear that $\epsilon = 0$ implies a uniform ambient temperature, for which the similarity solution has been reported by Cheng and Minkowycz.[4] For $\epsilon = 1$, it implies that the ambient temperature is the same as the wall temperature, therefore, no convection is expected. The series expressions for f and θ can be substituted into Eqs. (1) and (2), and the terms grouped as coefficients of like powers of ϵ are set equal to zero.

Local Nonsimilarity Method

Instead of expanding the series to include a higher order term, i.e., a fourth and possibly a fifth term, we opt for a locally nonsimilar solution.[5-7]

To begin with, we redefine the nondimensional normal coordinate [η in Eq. (5)] as the pseudosimilarity variable, and replace ϵ by ξ as the nonsimilarity variable.

At the second level of truncation, governing Eqs. (1) and (2) are transformed to

$$f' = \theta \tag{8}$$

$$\theta'' + \tfrac{1}{2} f\theta' = \lambda\xi f' + \lambda\xi[f'\phi - G\theta'] \tag{9}$$

$$G' = \phi \tag{10}$$

$$\phi'' + \frac{1}{2}f\phi' - \lambda f'\phi + \left(\lambda + \frac{1}{2}\right)G\theta' - \lambda f' - \lambda\xi G' + \lambda\xi[G\phi' - G'\phi] = \lambda\xi\left[f'\frac{\partial\phi}{\partial\xi} - \theta'\frac{\partial G}{\partial\xi}\right] \tag{11}$$

with the corresponding boundary conditions given by

$$f(\xi,0) = G(\xi,0) = 0, \qquad \theta(\xi,0) = 1 - \xi, \qquad \phi(\xi,0) = -1 \tag{12}$$

$$\theta(\xi,\infty) = 0, \qquad \phi(\xi,\infty) = 0 \tag{13}$$

where the primes refer to differentiation with respect to η. Auxiliary functions G and ϕ are defined by

$$G = \partial f/\partial\xi, \qquad \phi = \partial\theta/\partial\xi \tag{14}$$

Equations (8) and (10) have been simplified by integrating Eq. (1) once and invoking the boundary conditions at infinity [Eq. (4)].

To close the system of equations at the second level, terms involving $\partial G/\partial\xi$ and $\partial\phi/\partial\xi$ in Eq. (11) are deleted, and the whole set is treated as a coupled system of ordinary differential equations with ξ as a parameter.

Although the analysis can be further extended to include the third level of truncation, the computational cost forbids us to do so. Confidence on the solutions at the second level of truncation has stemmed from the successful application of this method to a similar problem by Chen and Eichhorn.[8]

Results and Discussion

The sets of ordinary differential equations derived in the last section, with their corresponding boundary conditions, are solved by numerical integration using the fourth-order Runge-Kutta method along with the shooting technique. This solution procedure and convergence criteria have been discussed in detail by Chen,[7] and so are omitted here for brevity.

Since a stable linearly stratified environment is encountered most frequently, we will place our attention on this specific case. To find the solutions at the second level of truncation, we found it necessary to place "infinity" very near $\eta = 0$ to start the calculation. "Infinity" was then increased as more and more refined estimates of the missing initial values were obtained. This computational scheme was also reported by Chen and Eichhorn.[8]

The heat-transfer coefficient, in terms of the Nusselt number, can be defined by

$$Nu = hx/k = \frac{qx}{(T_w - T_{\infty,o})k}$$

$$= -\theta'(\xi,0)Ra^{1/2}, \quad \text{for local nonsimilarity solution}$$

$$= -\left[\theta_0'(0) + \epsilon\theta_1'(0) + \epsilon^2\theta_2'(0) + \ldots\right]Ra^{1/2}, \quad \text{for series solutions} \tag{15}$$

The results can best be presented by the ratio of the heat-transfer coefficient for the thermally stratified case to that for the uniform case, shown in Fig. 2. As reported by Cheng and Minkowycz,[4] the heat-transfer coefficient for the uniform case is a constant and has a value of 0.4445. The integral solutions

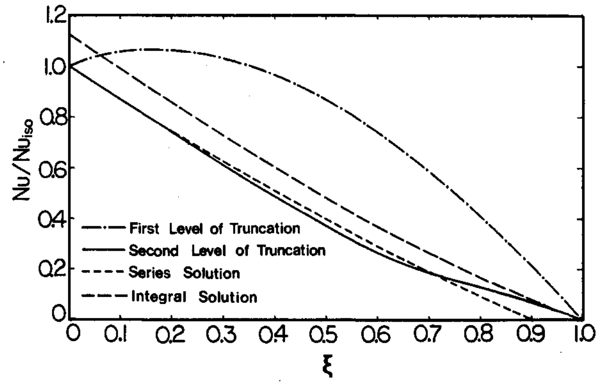

Fig. 2 Heat-transfer results for natural convection in a thermally stratified porous medium.

reported by Bejan[9] are also included in this figure for comparison.

It is observed that the solutions obtained by the series expansion agree very well with those obtained by the local nonsimilarity method. This is especially true for a small value of ϵ (or ξ). The discrepancy between these two solutions becomes noticeable when ξ is large. The integral method, on the other hand, always overpredicts the heat-transfer result. This discrepancy, however, can be minimized if a better temperature profile is assumed in the analysis. Nevertheless, all methods predict a reduced heat-transfer coefficient, due to the thermal stratification.

Acknowledgment

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